October 3, 2018

Calculus II SI Worksheet

Manipulating Integrals – Find the following indefinite integrals by algebraically rearranging their integrands

$$\int \frac{x^3+\sqrt{x}}{x^{\frac{3}{2}}}\,dx = \int \frac{x^3}{x^{\frac{3}{2}}} + \frac{\sqrt{x}}{x^{\frac{3}{2}}}\,dx = \int \frac{x^{\frac{6}{2}}}{x^{\frac{3}{2}}} + \frac{x^{\frac{1}{2}}}{x^{\frac{3}{2}}}\,dx = \int x^{\frac{3}{2}} + x^{-1}\,dx$$

$$= \boxed{\frac{2}{5}x^{\frac{5}{2}} + \ln x + C}$$

$$\int \frac{dx}{\sqrt{x^2+6x+16}} = \int \frac{dx}{\sqrt{(x+3)^2+7}} = \int \frac{du}{\sqrt{u^2+7}} = \operatorname{asinh}\left(\frac{u}{\sqrt{7}}\right)+C = \boxed{\operatorname{asinh}\left(\frac{x+3}{\sqrt{7}}\right)+C}$$

Complete the square:

$x^2+6x+16 = x^2+6x+9-9+16$

$= (x+3)^2+7$

$u = x+3$

$du = dx$

Integration by Parts – Evaluate the following integrals

$$\int x\sin x\,dx = uv - \int v\,du = -x\cos x + \int \cos x\,dx = \boxed{-x\cos x + \sin x + C}$$

$u = x \quad du = dx$

$v = -\cos x \quad dv = \sin x\,dx$

$$\int_1^2 \ln x\,dx = uv\Big|_1^2 - \int_1^2 v\,du = \ln(x)\,x\Big|_1^2 - \int_1^2 x\frac{1}{x}\,dx = \Big[x\ln(x) - x\Big]_1^2 = [2\ln(2)-2]-[\cancel{1\ln(1)}-1]$$

$$= 2\ln(2) - 2 + 1 = \boxed{2\ln(2)-1}$$

$u = \ln(x) \quad du = \frac{1}{x}dx$

$v = x \quad dv = dx$