Using the Ratio Test – Use the Ratio Test to determine whether the following series converge

1. $\sum_{k=1}^{\infty} \frac{(k+2)!}{k!}$

$a_k = \frac{(k+2)!}{k!} = \frac{\cancel{k!}(k+1)(k+2)}{\cancel{k!}} = (k+1)(k+2)$

$r = \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{\cancel{(k+2)}(k+3)}{(k+1)\cancel{(k+2)}} = \lim_{k\to\infty} \frac{\cancel{k}(1+\frac{3}{k})}{\cancel{k}(1+\frac{1}{k})}$

$= \frac{1+\cancel{3/\infty}^{0}}{1+\cancel{1/\infty}^{0}} = \boxed{1} \Rightarrow$ inconclusive

2. $\sum_{k=1}^{\infty} \frac{k!}{(k+1)!}$

$a_k = \frac{\cancel{k!}}{\cancel{k!}(k+1)} = \frac{1}{k+1}$

$r = \lim_{k\to\infty} \frac{1/k+2}{1/k+1} = \lim_{k\to\infty} \frac{k+1}{k+2} = \frac{1+0}{1+0} = \boxed{1} \Rightarrow$ inconclusive

Using the Root Test – Use the Root Test to determine whether the following series converge

3. $\sum_{k=1}^{\infty} \frac{2^k}{k^{10}}$

$\rho = \lim_{k\to\infty} \sqrt[k]{\frac{2^k}{k^{10}}} = \lim_{k\to\infty} \frac{2}{k^{\frac{10}{k}}} = \frac{2}{\lim_{k\to\infty} k^{\frac{10}{k}}} = \frac{2}{1} = \boxed{2}$ $\boxed{\rho > 1 \therefore \text{ series diverges}}$

$\lim_{k\to\infty} k^{\frac{10}{k}} = \lim_{k\to\infty} exp(\ln(k^{\frac{10}{k}})) = \lim_{k\to\infty} exp(\frac{10}{k}\ln(k)) = exp(10 \lim_{k\to\infty} \frac{\ln(k)}{k}) = exp(10 \lim_{k\to\infty} \frac{1}{k} \cdot \frac{1}{k^2}) = exp(0) = 1$

Using the Comparison Test – Determine whether the following series converge

4. $\sum_{k=1}^{\infty} \frac{k^3}{2k^4 - 1}$

Compare to $\sum_{k=1}^{\infty} \frac{1}{2k}$, which diverges by p-series test

$\frac{k^3}{2k^4-1} > \frac{1}{2k^4-1} > \frac{1}{2k^4}$

$\frac{k^3}{2k^4-1} > \frac{k^3}{2k^4} = \frac{1}{2k} \Rightarrow$ $\boxed{\text{This series diverges}}$

5. $\sum_{k=1}^{\infty} \frac{\ln k}{k^3}$

$\frac{\ln(k)}{k^3} < \frac{k}{k^3} = \frac{1}{k^2}$ which converges by p-series test

$\therefore$ This series converges

Alternating Series Test – Determine the following series converge or diverge

6. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$

$\lim_{k\to\infty} \frac{1}{k^2} = 0$ ✓

$\boxed{\text{This series converges}}$

7. $\sum_{k=1}^{\infty} \frac{(-1)^k \ln k}{k}$

$\lim_{k\to\infty} \frac{\ln(k)}{k} = 0$ ✓

$\boxed{\text{This series converges}}$