Calculus II SI Worksheet September 12, 2018

Knowledge Bank

You can use these "pieces" to evaluate other problems in the worksheet. If you get stuck, come back here!

$\frac{d}{dx}\sqrt{x+1} =$ $(x+1)^{-\frac{1}{2}}$

$\int_1^0 x^2 dx =$ $-\frac{1}{3}$

$\int_0^1 x^2 dx =$ $\frac{1}{3}$

Practice

Evaluating Definite Integrals – Evaluate the following definite integrals using the Fundamental Theorem of Calculus, Part 2

$\int_0^{10} (60x - 6x^2)dx =$ $30x^2 - 2x^3 \Big|_0^{10} = 30(10)^2 - 2(10^3) - 30(0)^2 + 2(0)^3 = 3000 - 20000$

$= \boxed{-17{,}000}$

$\int_0^{2\pi} 3\sin x\, dx =$ $3\left[-\cos x\right]\Big|_0^{2\pi} = 3\left[-\cos 2\pi + \cos 0\right] = 3\left[-1+1\right] = \boxed{0}$

$\int_{\frac{1}{16}}^{\frac{1}{4}} \frac{\sqrt{t}-1}{t} dt =$ $\int_{\frac{1}{16}}^{\frac{1}{4}} \frac{1}{\sqrt{t}} - \frac{1}{t}\, dt = 2\sqrt{t} - \ln t \Big|_{\frac{1}{16}}^{\frac{1}{4}} = \boxed{\frac{1}{2} - \ln 4}$

Derivatives of Integrals – Use Part 1 of the Fundamental Theorem to simplify the following expressions

$\frac{d}{dx}\int_1^x \sin^2 t\, dt =$ $\boxed{\sin^2 x}$

$\frac{d}{dx}\int_x^5 \sqrt{t^2+1}\, dt =$ $-\sqrt{x^2+1}$

$\frac{d}{dx}\int_0^{x^2} \cos t^2\, dt =$ $\frac{du}{dx}\int_0^u \cos t^2 dt \frac{d}{du} = \left(\frac{d}{du}\int_0^u \cos t^2 dt\right) 2x = \cos u^2 dx$ $\boxed{2x\cos x^4 + C}$

$u = x^2$

$du = 2x\, dx$

Average Value Equals Function Value – Find the point(s) on the interval $(0,1)$ at which $f(x) = 2x(1-x)$ equals its average value on $[0,1]$

$$\text{Mean value} = \frac{\int_a^b f(x)\,dx}{b-a} = \frac{\int_0^1 2x(1-x)\,dx}{1-0} = \int_0^1 2x - 2x^2\,dx = x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$= \frac{1}{3}$$

$$\frac{1}{3} = 2x(1-x) \Rightarrow \frac{1}{6} = x - x^2 \Rightarrow \boxed{x = \frac{1}{2} - \frac{1}{2\sqrt{3}},\ \frac{1}{2} + \frac{1}{2\sqrt{3}}}$$

The Substitution Method – Find the following indefinite integrals

$$\int x^4(x^5+6)^9dx = \int \cancel{x^4}\, u^9 \frac{du}{5\cancel{x^4}} = \frac{1}{5}\int u^9 du = \frac{1}{5}\left[\frac{1}{10}u^{10} + C\right] = \boxed{\frac{1}{50}(x^5+6)^{10} + C}$$

$u = x^5+6$

$du = 5x^4\,dx$

$$\int \cos^3 x \sin x\ dx = \int u^3 \cancel{\sin x} \frac{du}{-\cancel{\sin x}} = -\int u^3 du \quad -\frac{1}{4}u^4 + C = \boxed{-\frac{1}{4}\cos^4 x + C}$$

$u = \cos(x)$

$du = -\sin x\, dx$

$$\int \frac{x}{\sqrt{x+1}}dx = \int \frac{u-1}{\sqrt{u}}du = \int \sqrt{u} - \frac{1}{\sqrt{u}}du = \frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + C$$

$u = x+1$

$du = dx$

$$= \boxed{\frac{2}{3}(x+1)^{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}} + C}$$