Calculus II SI Worksheet October 24, 2018

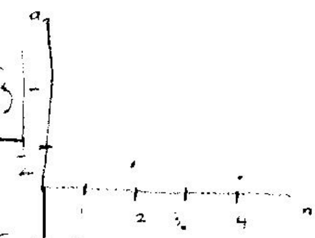
Explicit Sequence Formulas – Use the explicit formula for $\{a_n\}_{n=1}^{\infty}$ to write the first four terms of each sequence and sketch a graph of the sequence.

1. $a_n = \dfrac{1}{2^n}$

$a_n = \left\{\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \dots\right\} = \boxed{\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\right\}}$

2. $a_n = \dfrac{(-1)^n n}{n^2+1}$

$a_n = \left\{\frac{(-1)^1\cdot 1}{1^2+1}, \frac{(-1)^2\cdot 2}{2^2+1}, \frac{(-1)^3\cdot 3}{3^2+1}, \frac{(-1)^4\cdot 4}{4^2+1}\right\} = \boxed{\left\{-\frac{1}{2}, \frac{2}{5}, -\frac{3}{10}, \frac{4}{17}, \dots\right\}}$



Working with Sequences – For the following sequences, find the next two terms of the sequence, find a recurrence relation that generates the sequence, and then find an explicit formula for the n^{th} term of the sequence.

3. $\{a_n\} = \{-2, 5, 12, 19, \dots\}$

$\{a_n\}_{n=0}^{\infty} = \{-2, 5, 12, 19, 26, 33, \dots\} = \boxed{7n-2}$

4. $\{b_n\} = \{3, 6, 12, 24, 48, \dots\}$

$= \{3, 6, 12, 24, 48, 96, 192, \dots\}$

$= \boxed{3(2)^{n-1}}$

Limits of Sequences – Write and graph the first tour terms of each sequence and conjecture about its limit. If the limit appears to diverge, informally prove that it does indeed diverge.

5. $\left\{\dfrac{(-1)^n}{n^2+1}\right\}_{n=1}^{\infty} = \left\{\frac{(-1)^1}{1^2+1}, \frac{(-1)^2}{2^2+1}, \frac{(-1)^3}{3^2+1}, \frac{(-1)^4}{4^2+1}\right\} = \left\{\frac{-1}{2}, \frac{1}{5}, \frac{-1}{10}, \frac{1}{17}, \dots\right\}$

$\boxed{\lim_{n\to\infty} a_n = 0}$

6. $\{\cos \pi n\}_{n=1}^{\infty}$ $= \{\cos(\pi), \cos(2\pi), \cos(3\pi), \cos(4\pi)\} = \{-1, 1, -1, 1\}$

cos oscilates, ∴ this sequence diverges

7. $\{a_n\}_{n=1}^{\infty}$ where $a_{n+1} = -2a_n$, $a_1 = 1$

$= \{1, -2, 4, -8\}$

magnitude is always increasing ∴ this sequence diverges

Analytical Limits of Sequences – Find the limit of the following sequences by evaluating as a limit, or by using a theorem in your textbook

8. $a_n = \sin \dfrac{\pi n}{2}$

sin oscilates, ∴ diverges

9. $a_n = (-1)^n \sqrt[n]{n}$

$\lim_{n\to\infty} \sqrt[n]{b} = \infty$, $\lim_{n\to\infty} n = \infty$ ∴ $\lim_{n\to\infty} a_n = \infty$

10. $a_n = \dfrac{n!}{n^n}$

n^n increases faster than $n!$, and by Theorem 8.6

$\lim_{n\to\infty} a_n = \boxed{0}$