Calculus II SI Worksheet October 31, 2018

**1. Squeeze Theorem** – Find the limit of the sequence $a_n = \frac{\cos n}{n^2+1}$

for all $n \geq 1$

$$\underbrace{-\frac{1}{n^2+1}}_{z_n} \leq \frac{\cos(n)}{n^2+1} \leq \underbrace{\frac{1}{n^2+1}}_{y_n}$$

$$\lim_{n\to\infty} z_n = \lim_{n\to\infty} -\frac{1}{n^2+1} = 0$$

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} \frac{1}{n^2+1} = 0$$

By squeeze theorum, $z_n \leq a_n \leq y_n$ $\therefore$ $\lim_{n\to\infty} z_n \leq \lim_{n\to\infty} a_n \leq \lim_{n\to\infty} y_n$

$$\Rightarrow \boxed{\lim_{n\to\infty} a_n = 0}$$

**2. Super Computing Geek-out** – Some friends and I were having a debate on whether classical computing could be used to appropriately map rare genome-to-phenome associations (GWAS). The number of entries that must be processed in GWAS is made up of a matrix of the square of the number of individuals in a population added to a matrix of the number of individuals multiplied by the number of known base pair differences ($g_n = n^2 + Mn$). Moore's Law states that computing power will double every two years ($c_n = K2^n$). For a large dataset and computer far in the future ( $\lim_{n\to\infty}$ ), will computing power or the number of records dominate?

For $g_n$, $n^2$ grows faster than $Mn$

Comparing $n^2$ to $2^n$, by theorem 8.6, $2^n$ grows faster $\therefore$ computing power will be sufficent.

**Geometric Series** – Evaluate each geometric series or state that it diverges

**3.** $\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k = \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \boxed{\frac{4}{3}}$

$a = \left(\frac{1}{4}\right)^0 = 1$

$r = \frac{1}{4}$

**4.** $\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \left[\frac{1}{1} - \cancel{\frac{1}{2}}\right] + \left[\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right] + \left[\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right] + \dots + \left[\cancel{\frac{1}{n}} - \frac{1}{1+n}\right] = 1 - \frac{1}{1+n}$

$$\frac{1}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1} \Rightarrow \frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

$$\lim_{k\to\infty} 1 - \frac{1}{1+k} = \boxed{1}$$

**Review** – Solve the following problems

**5.** Find $\int \frac{x^2-x}{(x-2)(x-3)^2}dx$ $= \int \frac{2}{x-2} - \frac{1}{x-3} + \frac{6}{(x-3)^2}dx = \boxed{2\ln(x-2) - \ln(x-3) - \frac{6}{x-3} + C}$

$\frac{x^2-x}{(x-2)(x-3)^2} = \frac{A}{x-2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$\Rightarrow x^2-x = A(x-3)^2 + B(x-2)(x-3) + C(x-2)$

let $x=3 \Rightarrow 6 = C$

let $x=2 \Rightarrow 2 = A$

let $x=0 \Rightarrow 0 = 18+6B-12 \Rightarrow B=-1$

**6.** Evaluate $\int_0^\infty e^{-3x}dx$

$= \lim_{b\to\infty} \int_0^b e^{-3x}dx = \lim_{b\to\infty}\left(-\frac{1}{3}e^{-3x}\right)\Big|_0^b = \lim_{b\to\infty}\frac{1}{3}\left(1-\frac{1}{e^{3b}}\right) = \frac{1}{3}\left(1-\frac{1}{\infty}\right) = \boxed{\frac{1}{3}}$

**7.** Solve for $y$ such that $y'(t) = 2.5y$, and $y(0) = 3.2$

$\frac{dy}{y} = 2.5dt \Rightarrow \ln(y) = 2.5t + C \Rightarrow y = e^{2.5t} + C$

$y(0) = 3.2 = e^{2.5(0)} + C \Rightarrow C = 3.2$

$\Rightarrow \boxed{y(t) = e^{2.5t} + 3.2}$

**8.** Find the limit of $\left\{a_n = \frac{4n^3}{n^3+1}\right\}_{n=1}^{\infty}$

$\lim_{n\to\infty}\frac{4n^3}{n^3+1} = \lim_{n\to\infty}\frac{\cancel{n^3}}{\cancel{n^3}}\frac{4}{1+\frac{1}{n^3}} = \frac{4}{1+\frac{1}{\infty^3}} = \boxed{4}$