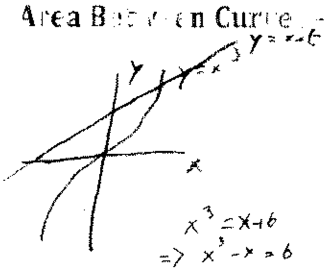
Area Between Curves - Find the area of the region R bounded by the graphs of $y = x^3$, $y = x + 6$, and the x-axis.

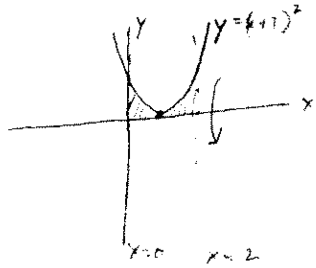


$$A = \int_{-6}^{0} x+6\,dx + \int_{0}^{2} x+6-x^3\,dx = \frac{1}{2}x^2+6x\Big|_{-6}^{0} + \frac{1}{2}x^2+6x-\frac{1}{4}x^4\Big|_{0}^{2}$$

$x^3 = x+6$
$\Rightarrow x^3 - x = 6$
$x = 2$

$$= \frac{1}{2}0^2 + 6\cdot 0 - \frac{1}{2}(36) - (-36) + \frac{1}{2}4 + 12 - \frac{1}{4}16 - \frac{1}{2}0^2 + 6.0 + \frac{1}{4}0^4 = \boxed{64}$$

The Disk Method - Let R be the region bounded by the curve $f(x) = (x+1)^2$, the x-axis, and the lines $x = 0$ and $x = 2$. Find the volume of the solid of revolution obtained by revolving R about the x-axis.



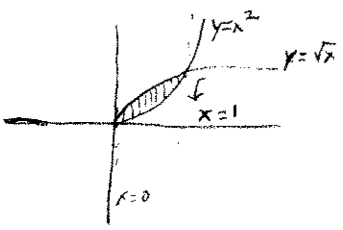
$u = x+1 \quad du = dx$
$u(0) = 1, \quad u(2) = 3$

$$V = \pi r^2 l = \int_0^2 \pi (x+1)^4 dx = \pi\int_0^2 (x+1)^4 dx$$

$$= \pi\int_1^3 u^4 du = \pi\left[\frac{1}{4}u^5\right]\Big|_1^3 = \pi\left[\frac{1}{4}3^5 - \frac{1}{4}1^5\right] = \pi\left[\frac{243}{4} - \frac{1}{4}\right]$$

$$= \pi\left[\frac{242}{4}\right] = \boxed{\frac{121}{2}\pi}$$

The Washer Method - The region R is bounded by the graphs of $f(x) = \sqrt{x}$ and $g(x) = x^2$ between $x = 0$ and $x = 1$. What is the volume of the solid that results when R is revolved about the x-axis?

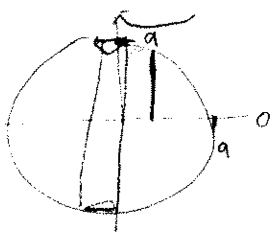


$$V = \int_0^1 \pi r^2 dx = \pi\int_0^1 (x^{\frac{1}{2}} - x^2)^2 dx = \pi\int_0^1 x - 2x^{\frac{5}{2}} + x^4\,dx$$

$$= \pi\left[\frac{1}{2}x^2 - \frac{4}{7}x^{\frac{7}{2}} + \frac{1}{5}x^5\right]\Big|_0^1 = \pi\left[\frac{1}{2} - \frac{4}{7} + \frac{1}{5}\right]$$

$$= \pi\left[\frac{1\cdot 7\cdot 5}{70\ \cancel{2\cdot 7\cdot 5}} - \frac{4\cdot 2\cdot 5}{70} + \frac{1\cdot 2\cdot 7}{70}\right] = \frac{35+40+14}{70}\pi = \boxed{\frac{89}{70}\pi}$$

The Shell Method - A cylindrical hole with radius r is drilled symmetrically through the center of a sphere with radius a, where $0 \le r \le a$. What is the volume of the remaining material?



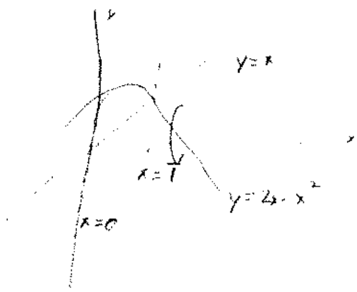
$$V = 2\int_r^a 2\pi\sqrt{a^2-x^2}\,dx$$

$x^2 + y^2 = a^2$
$\Rightarrow y = \sqrt{a^2 - x^2}$

Calculus II SI Worksheet

September 12, 2018

Volume by Which Method? – The region R is bounded by the graphs of $f(x) = 2x - x^2$ and $g(x) = x$ on the interval [0,1]. Use the washer method and the shell method to find the volume of the solid formed when R is revolved about the x-axis.



$2x - x^2 = x$

$x = 1 \quad 2 - 1 = 1$

$y = 2x - x^2 = 2(1) - (1) = 1$

Washer Method

$$V = \pi\int_0^1 r^2\,dx = \pi\int_0^1 [(2x-x^2)-x]^2\,dx$$

$$= \pi\int_0^1 (x-x^2)^2\,dx = \pi\int_0^1 x^2 - 2x^3 + x^4\,dx$$

$$= \pi\left[\frac{1}{3}x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5\right]\Big|_0^1$$

$$= \pi\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] = \pi\left[\frac{10 - 15 + 3}{30}\right] = -\pi\frac{2}{30} = \boxed{-\pi\frac{1}{15}}$$

Shell Method

$$V = \int 2\pi r\,dy = \int_0^1 2\pi(y - 1 - \sqrt{1-y})\,dy$$

$2x - x^2 = y \Rightarrow$

$x = 1 - \sqrt{1-y}$