**Compressing a Spring** – Suppose a force of 10 N is required to stretch a spring 0.1 m from is equilibrium position and hold it in that position.

**a.** Assuming that the spring obeys Hooke's Law, find the spring constant *k*.
**b.** How much work is needed to compress the spring 0.5 m from its equilibrium position?
**c.** How much work is needed to stretch the spring 0.25 m from its equilibrium position?
**d.** How much additional work is required to stretch the spring 0.25 m from its equilibrium position?

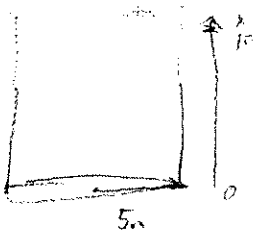
a. $k = \frac{F}{\Delta x} = \frac{10\,N}{0.1\,m} = \boxed{100\ N/m}$

b. $W = \int_a^b F(x)\,dx = \int_0^{\frac{1}{2}} kx\,dx = \int_0^{\frac{1}{2}} 100x\,dx = 50x^2\Big|_0^{\frac{1}{2}} = \boxed{12\frac{1}{2}}$

c. $\int_0^{\frac{1}{4}} 100x\,dx = 50x^2\Big|_0^{\frac{1}{4}} = \boxed{3\frac{1}{8}}$

d. $\int_{0.1}^{0.35} 100x\,dx = \boxed{5.625}$

**Pumping Water** – How much work is needed to pump all the water our of a cylindrical tank with a height of 10 m and a radius of 5 m? The water is pumped to an outflow pipe 15 m above the bottom of the tank.




$$W = \int_a^b F(x)\,dx = \int_a^b mg\,dx = \int_a^b \rho g A D\,dx$$

$$= \int_0^{10} \rho g (5^2)\pi (15-x)\,dx = 25\rho g\pi \int_0^{10} (15-x)\,dx$$

$$= 25\rho g\pi \left[15x - \frac{1}{2}x^2\right]\Big|_0^{10} = (25)(1000)(9.8)\pi\left[150 - \frac{1}{2}100\right]$$

$$= (25)(1000)(9.8)\pi(100) \approx \boxed{7.7\times10^7\ J}$$

**REVIEW** – Solve the following problems

Evaluate $\int_{-2}^{3} x^2 - x - 6\,dx$

$\int_{-2}^{3} x^2 - x - 6\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x \Big|_{-2}^{3} = \boxed{\frac{-125}{6}}$

Find the average value of $f(x) = \cos x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\bar{f} = \int_a^b f(x)\frac{1}{b-a} = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = \frac{1}{\pi}\left[\sin x\right]\Big|_{-\pi/2}^{\pi/2} = \frac{1}{\pi}(1+1) = \boxed{\frac{2}{\pi}}$

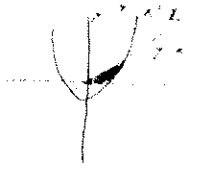
Evaluate $\int 2x(x^2-1)^{99}\,dx$

$u = x^2 - 1$

$du = 2x\,dx$

$\int 2x(x^2-1)^{99}\,dx = \int 2x\,u^{99}\frac{du}{2x} = \int u^{99}\,du \qquad \boxed{\frac{1}{100}u^{100} + C}$

Find the area bounded between $f(x) = x$ and $g(x) = x^2 - 2$



$x = x^2 - 2$

$x = 2$

$x = -1$

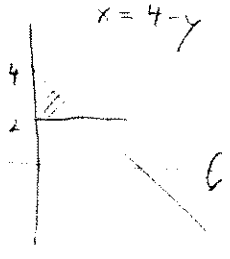
$\int_{-1}^{2} x - x^2 + 2\,dx = \frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x \Big|_{-1}^{2} = \boxed{\frac{9}{2}}$

Find the volume of the solid formed by revolving the curve $f(x) = e^{-x}$ around the $x$-axis on $(0, \ln 4)$

$V = \int_0^{\ln(4)} \pi e^{-2x}\,dx = -\frac{\pi}{2}e^{-2x}\Big|_0^{\ln(4)} = \boxed{\frac{15\pi}{32}}$

Find the volume of the solid formed by revolving the area between the curves $f(x) = 4 - x$ and $g(x) = 2$ around the $x$-axis

$x = 4 - y$




$V = 2\pi\int_2^4 y(4-y) = 2\pi\left[2y^2 - \frac{y^3}{3}\right]\Big|_2^4 = \boxed{\frac{32\pi}{3}}$