Operations on Power Series – Recall that $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$. Using this series, find the power series representation of the following expressions.

1. $\frac{x^5}{1-x}$

2. $\frac{1}{1+x^2}$

3. $\frac{d}{dx}\frac{1}{1-x}$

Maclaurin Series – Find the Maclaurin series and interval of convergence for each of the following functions.

4. $f(x) = \cos x$

5. $g(x) = e^{2x}$

6. Evaluating a Limit by Taylor Series – Evaluate $\lim_{x\to\infty} 6x^5 \sin\frac{1}{x} - 6x^4 + x^2$

REVIEW

Convergence of Series – Pick a test and determine if each of the following series converges. If it is an alternating series, determine if the convergence is absolute or conditional.

7. $\sum_{k=0}^{\infty} \frac{k}{2k+1}$

8. $\sum_{k=1}^{\infty} \frac{k^2}{4^k}$

9. $\sum_{k=1}^{\infty} \frac{\cos k}{k^3}$

10. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{\frac{3}{2}}}$

Taylor Polynomials – Find the 2nd-order Taylor polynomial centered at 0 for the following functions.

11. $f(x) = \ln(x - 1)$

12. $g(x) = \tan x$

Estimating Real Numbers – Estimate the value of the following numbers using a 2nd-order Taylor polynomial of your choice. Center the polynomial at the closest known value of the function you chose.

13. $(7.5)^{\frac{1}{3}}$

14. $\sqrt{3.9}$