**Powers of sine or cosine** – Evaluate the following integrals

$u = \sin(x),\ du = \cos(x)dx$

1. $\int \cos^5 x \, dx = \int \cos^4(x)\cos(x)\,dx = \int (1-\sin^2(x))^2 \cos(x)\,dx = \int (1-u^2)^2 \cancel{\cos(x)}\frac{du}{\cancel{\cos(x)}} = \int 1 - 2u^2 + u^4 \, du$

$= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C = \boxed{\sin(x) - \frac{2}{3}\sin^3(x) + \frac{1}{5}\sin^5(x) + C}$

$u = \cos(x)\quad du = -\sin(x)dx$

2. $\int \sin^3 x \, dx = \int \sin^2(x)\sin(x)\,dx = \int (1-\cos^2(x))\sin(x)\,dx = -\int (1-u^2)\cancel{\sin(x)}\frac{du}{\cancel{\sin(x)}} = -\int 1 - u^2 \, du$

$= \frac{1}{3}u^3 - u + C = \boxed{\frac{1}{3}\cos^3(x) - \cos(x) + C}$

3. $\int \sin^4 x \, dx = \int \left(\frac{1-\cos(2x)}{2}\right)^2 dx = \frac{1}{4}\int (1-\cos(2x))^2 dx = \frac{1}{4}\int 1 - 2\cos(2x) + \cos^2(2x)\, dx$

$= \frac{1}{4}\int 1 - 2\cos(2x) + \frac{1+\cos(4x)}{2}\,dx = \frac{1}{4}\int \frac{3}{2} - 2\cos(2x) + \frac{1}{2}\cos(4x)\,dx = \frac{1}{4}\left[\frac{3}{2}x - \sin(2x) + \frac{1}{8}\sin(4x) + C\right]$

$= \boxed{\frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C}$

4. $\int \sin^4 x \cos^2 x \, dx = \int \left(\frac{1-\cos(2x)}{2}\right)^2\left(\frac{1+\cos(2x)}{2}\right)dx = \frac{1}{8}\int (1-\cos(2x))^2(1+\cos(2x))\,dx$

$= \frac{1}{8}\int 1 - \cos(2x) - \cos^2(2x) + \cos^3(2x)\,dx = \frac{1}{8}\int 1 - \cos(2x) - \frac{1}{2} - \cos(4x)\,dx + \frac{1}{8}\int (1-\sin^2(2x))\cos(2x)\,dx$

$u = \sin(2x),\ du = 2\cos(2x)dx$

$= \frac{1}{8}x - \frac{1}{16}\sin(2x) - \frac{1}{16}x - \frac{1}{32}\sin(4x) + \frac{1}{16}\int (1-u^2)\,du = \boxed{\frac{1}{16}x - \frac{1}{64}\sin(4x) - \frac{1}{48}\sin^3(2x) + C}$

5. $\int \frac{\sin^3 x}{\cos^2 x}\,dx = \int \frac{\sin(x)\sin^2(x)}{\cos^2(x)}\,dx = \int \frac{\sin(x)(1-\cos^2(x))}{\cos^2(x)}\,dx = -\int \frac{1-u^2}{u^2}\,du = -\int \frac{1}{u^2} - 1\,du$

$u = \cos(x)$

$du = -\sin(x)dx$

$= u + \frac{1}{u} + C = \boxed{\cos(x) + \sec(x) + C}$

$u = \sin(x),\ du = \cos(x)dx$

6. $\int \sin^3 x \cos^3 x \, dx = \int \sin^3(x)\cos(x)(1-\sin^2(x))\,dx = \int u^3(1-u^2)\,du = \int u^3 - u^5\,du = \frac{1}{4}u^4 - \frac{1}{6}u^6 + C$

$= \boxed{\frac{1}{4}\sin^4(x) - \frac{1}{6}\sin^6(x) + C}$

Calculus II SI Worksheet

October 10, 2018

**Partial Fractions** – Evaluate the following integrals

7. $\int \frac{3x}{x^2+2x-8}\,dx = \int \frac{1}{x-2} + \frac{2}{x+4}\,dx = \ln|x-2| + \ln|x+4| + C = \boxed{\ln((x-2)(x+4)) + C}$

$\frac{3x}{x^2+2x-8} = \frac{3x}{(x-2)(x+4)} = \frac{A}{x-2} + \frac{B}{x+4} \Rightarrow 3x = A(x+4) + B(x-2)$

set $x=-4 \Rightarrow -12 = \cancel{A(0)} + B(-6) \Rightarrow B = 2$

set $x = 2 \Rightarrow 6 = A(6) + \cancel{B(0)} \Rightarrow A = 1$

8. $\int \frac{3x^2+7x-2}{x^3-x^2-2x}\,dx =$

$\int \frac{1}{x} - \frac{2}{x+1} + \frac{4}{x-2}\,dx = \boxed{\ln(x) - 2\ln(x+1) + 4\ln(x-2) + C}$

$\frac{3x^2+7x-2}{x^3-x^2-2x} = \frac{3x^2+7x-2}{x(x+1)(x-2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-2} \Rightarrow 3x^2+7x-2 = A(x+1)(x-2) + Bx(x-2) + Cx(x+1) = (A+B+C)x^2 - (-A-2B+C)x - 2A$

$\Rightarrow \begin{matrix} A+B+C=3 \\ -A-2B+C=7 \\ -2A=-2 \end{matrix} \Rightarrow \begin{matrix} A=1 \\ B=-2 \\ C=8 \end{matrix}$

9. $\int \frac{3x^2+2x+5}{(x-1)(x^2-x-20)}\,dx =$

$\int -\frac{1}{(-16)(x-1)} + \frac{5}{2(x-5)} + \frac{1}{x+4}\,dx = \boxed{-\frac{1}{2}\ln|x-1| + \frac{5}{2}\ln|x-5| + \ln|x+4| + C}$

$\frac{3x^2+2x+5}{(x-1)(x^2-x-20)} = \frac{3x^2+2x+5}{(x-1)(x-5)(x+4)} = \frac{A}{x-1} + \frac{B}{x-5} + \frac{C}{x+4} \Rightarrow 3x^2+2x+5 = A(x-5)(x+4) + B(x-1)(x+4) + C(x-1)(x-5)$

set $x=1 \Rightarrow 10 = -20A \Rightarrow A = -\frac{1}{2}$ set $x=-4 \Rightarrow 45 = 45 \Rightarrow C = 1$

set $x=5 \Rightarrow 90 = 36B \Rightarrow B = \frac{5}{2}$

10. $\int \frac{5x^2-3x+2}{x^3-2x^2}\,dx = \int \frac{1}{x} - \frac{1}{x^2} + \frac{3}{x-2}\,dx = \ln|x| + \frac{1}{x} + 3\ln(x-2) + C$

$\frac{5x^2-3x+2}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} \Rightarrow 5x^2-3x+2 = Ax(x-2) + B(x-2) + Cx^2$

set $x=2 \Rightarrow 12 = C(4) \Rightarrow C = 3$

set $x=0 \Rightarrow 2 = -2B \Rightarrow B = -1$

set $x=1 \Rightarrow 4 = -A + 1 + 3 \Rightarrow A = 1$

11. $\int \frac{10}{(x-2)^2(x^2+2x+2)}\,dx =$

$\frac{10}{(x-2)^2(x^2+2x+2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+2x+2}$ [illegible]

$\Rightarrow 10 = A(x-2)(x^2+2x+2) + B(x^2+2x+2) + (Cx+D)(x-2)^2$